Pramana – J. Phys. (2016) 87: 52
DOI 10.1007/s12043-016-1242-6

Root mean square radii of heavy flavoured mesons in a quantum chromodynamics potential model

TAPASHI DAS[1,*] and D K CHOUDHURY[1,2]

[1]Department of Physics, Gauhati University, Guwahati 781 014, India
[2]Physics Academy of North-East, Guwahati 781 014, India
*Corresponding author. E-mail: t4tapashi@gmail.com

MS received 11 April 2015; revised 15 July 2015; accepted 16 December 2015; published online 9 September 2016

Abstract. We report the results of root mean square (r.m.s.) radii of heavy flavoured mesons in a QCD model with the potential $V(r) = -(4\alpha_s/3r) + br + c$. As the potential is not analytically solvable, we first obtain the results in the absence of confinement and Coulomb terms respectively. Confinement and Coulomb effects are then introduced successively in the approach using the Dalgarno's method of perturbation. We explicitly consider the following two quantum mechanical aspects in the analysis: (a) The scale factor c in the potential should not effect the wave function of the system even while applying the perturbation theory. (b) Choice of perturbative piece of the Hamiltonian (confinement or linear) should determine the effective radial separation between the quarks and antiquarks. The results are then compared with the available theoretical values of r.m.s. radii.

Keywords. Quantum chromodynamics; Dalgarno's method; r.m.s. radius; charge radius.

PACS Nos 12.38.−t; 12.39.Pn

1. Introduction

In the past four decades, quantum chromodynamics (QCD) [1] has been established as the successful theory of strong interaction. In the soft region, where field theoretical perturbative methods do not work, lattice QCD [2] has emerged as the most promising approach. However, analytically approachable QCD-based phenomenological models [3,4] gained important roles in capturing the salient features of QCD in this regime, since the pioneering work of De Rujula Georgi and Glashow [5].

In this spirit, a QCD-inspired potential model has been pursued in refs [6–11] in recent years, with considerable phenomenological success. The results include static and dynamical properties of heavy flavoured mesons, such as their form factors, masses, decay constants as well as Isgur–Wise function [12] describing semileptonic decays.

The linear cum Coulomb potential is not exactly analytically solvable. We therefore applied Dalgarno's method [13,14] to it. In such an approach, one has two options: Coulomb or linear term as perturbation. However, while using perturbative method, one should consider two aspects of quantum mechanics: (a) The scale factor c in the potential should not affect the wave function of the meson even while using perturbation theory to be compatible with quantum mechanical expectation. (b) The specific choice of perturbative piece (Coulomb or linear) should determine the perturbatively compatible effective radial separation between the quark and the antiquark.

In the present work, we shall show that only in the short distance range ($0 < r < r^{\text{short}}$), linear potential is perturbatively compatible, while for the alternative choice (Coulomb as perturbation), the corresponding range belongs to large distance ($r^{\text{long}} < r < \infty$). The exact magnitudes of r^{short} and r^{long} have explicit dependence on strong coupling constant α_s and the confinement parameter b. In previous analyses [6–11], the two features (a) and (b) were overlooked.

The aim of the present paper is to outline the new features of the improved version of the model and present the prediction for the r.m.s. radii of pseudoscalar heavy flavoured mesons (both heavy–light and heavy–heavy). Comparison is then made with those of the other models available in literature.

In §2, the formalism is outlined, while in §3 the results are summarized. Section 4 contains the conclusion and comments.

2. Formalism

2.1 *Definition of r.m.s. radius*

The r.m.s. radius [15,16] of the bound state of the quark and antiquark like meson is defined as

$$\langle r^2\rangle = \int_0^\infty r^2[\psi(r)]^2 \mathrm{d}r, \tag{1}$$

having radial wave function $\psi(r)$.

2.2 *QCD potentials and the corresponding wave functions*

QCD potential between a quark and an antiquark has been one of the first important ingredients of the phenomenological models to be studied in quantum physics.

The Schrödinger equation describing the quark–antiquark bound state is

$$-\frac{\hbar^2}{2m}\nabla^2\psi(r) + (E-V)\psi(r) = 0. \tag{2}$$

The standard QCD potential is defined as [17]

$$V(r) = -\frac{4\alpha_s}{3r} + br + c, \tag{3}$$

where r is the interquark distance and c is a constant scale factor to make it comparable with the data.

The QCD potential is based on two important facts of QCD: asymptotic freedom and confinement. For mesons, the one-gluon exchange contribution between q and $\bar{q}$ is given by

$$V_1 = -\frac{4\alpha_s}{3r}. \tag{4}$$

$-(4/3)$ is due to the colour factor and α_s is the strong coupling constant.

The wave function corresponding to the potential (4) including relativistic effect [18,19] is

$$\psi^0(r) = \frac{1}{\sqrt{\pi a_0^3}} \mathrm{e}^{-r/a_0}\left(\frac{r}{a_0}\right)^{-\epsilon}, \tag{5}$$

where

$$a_0 = \left(\frac{4}{3}\mu\alpha_s\right)^{-1}, \tag{6}$$

$$\mu = \frac{m_1 m_2}{m_1+m_2}. \tag{7}$$

m_1 and m_2 are the masses of quark and antiquark, respectively, μ is the reduced mass of the mesons and

$$\epsilon = 1-\sqrt{1-\left(\frac{4}{3}\alpha_s\right)^2}, \tag{8}$$

is the relativistic effect due to the Dirac modification factor.

The linear form for the long-range part of the QCD potential is

$$V_2(r) = br, \tag{9}$$

where b is the confinement parameter. Phenomenologically, $b = 0.183$ GeV^2 [20].

For a meson, the wave function of the bound quark–antiquark state with this potential (9) is

$$\psi^0(r) = \frac{N}{r} A_i[Br - D]\left(\frac{r}{a_0}\right)^{-\epsilon}, \tag{10}$$

where

$$B = (2\mu b)^{-1/3} \tag{11}$$

and

$$D = \left(\frac{9\pi}{8}\right)^{2/3}. \tag{12}$$

Here, N is the normalization constant and $A_i[r]$ is the Airy function [10,21].

Potential (3) is not analytically solvable. We make two choices: Choice I: $-(4\alpha_s/3r)+c$ as the parent and br as the perturbation and Choice II: $br+c$ as the parent and $-(4\alpha_s/3r)$ as the perturbation.

We use the Dalgarno's method of perturbation to construct the wave function. It is well known in quantum mechanics that a constant term c in the potential can, at best, shift the energy scale, but should not perturb the wave function. This important point was overlooked in earlier publications [6–11] on the subject. The present work takes this into account and removes the limitation by Choice I and II, respectively, to make the perturbed component of the wave function c independent.

From the perturbation conditions we have

For Choice I:

$$-\frac{4\alpha_s}{3r} + c > br. \tag{13}$$

For Choice II:

$$br + c > -\frac{4\alpha_s}{3r}. \tag{14}$$

From (13) and (14), we can find the bounds on r upto which Choices I and II are valid. While Choice I gives the cut-off on the short distance $r_{\max}^{\text{short}}$, Choice II gives the cut-off on the long distance $r_{\min}^{\text{long}}$.

The wave function for Choice I is

$$\psi^{\text{total}}(r) = \frac{N'}{\sqrt{\pi a_0^3}}\left[1-\frac{1}{2}\mu b a_0 r^2\right]\mathrm{e}^{-(r/a_0)}\left(\frac{r}{a_0}\right)^{-\epsilon}, \tag{15}$$

which is identical to eq. (25) of ref. [6]. The later version of the wave function (eq. (4) of ref. [7]) does not conform to the quantum mechanical idea. N' is the normalization constant.

The wave function for Choice II is [10]

$$\psi^{\text{total}}(r) = N''\left[\left[\frac{1}{2\sqrt{\pi r}}\right]A_i[Br-D] -\frac{4}{3}\alpha_{\text{s}}(A_0 r^{-1}+A_1 r^1+A_2)\right]\left(\frac{r}{a_0}\right)^{-\epsilon}, \tag{16}$$

where N'' is the normalization constant. The coefficients A_0, A_1, A_2 are the same as eqs (17)–(19) of ref. [10], except that c will not appear here as we are considering c in the parent part of the Hamiltonian.

2.3 *Derivation of perturbative limits $r_{\max}^{\text{short}}$ and $r_{\min}^{\text{long}}$ and constraint on the scale factor c: The improved perturbative approach*

(a) *For Coulomb term as the parent and linear term as the perturbation*

$$-\frac{4\alpha_{\text{s}}}{3r}+c > br,$$

$$\left(-\frac{4\alpha_{\text{s}}}{3r}+c\right)^2 > (br)^2. \tag{17}$$

It leads to four possibilities:

$$c > \frac{4\alpha_{\text{s}}}{3r} - br \tag{18}$$

and

$$c > \frac{4\alpha_{\text{s}}}{3r} + br \tag{19}$$

or

$$c < \frac{4\alpha_{\text{s}}}{3r} - br \tag{20}$$

and

$$c < \frac{4\alpha_{\text{s}}}{3r} + br. \tag{21}$$

As (18) is a special case of (19), and (21) is a special case of (20), we consider (19) and (20) as the alternative plausible perturbative conditions for c. Inequality (19) can be read as

$$br^2 - cr + \frac{4\alpha_{\text{s}}}{3} < 0.$$

We consider

$$br^2 - cr + \frac{4\alpha_{\text{s}}}{3} = 0,$$

$$r_1^{\text{short}} = \frac{c+\sqrt{c^2-(16\alpha_{\text{s}}/3)b}}{2b} \tag{22}$$

and

$$r_2^{\text{short}} = \frac{c-\sqrt{c^2-(16\alpha_{\text{s}}/3)b}}{2b}. \tag{23}$$

Similarly, (20) can be read as

$$br^2 + cr - \frac{4\alpha_{\text{s}}}{3} < 0,$$

which saturates at

$$r_3^{\text{short}} = \frac{-c+\sqrt{c^2+(16\alpha_{\text{s}}/3)b}}{2b} \tag{24}$$

and

$$r_4^{\text{short}} = \frac{-c-\sqrt{c^2+(16\alpha_{\text{s}}/3)b}}{2b}. \tag{25}$$

(b) *For linear term as the parent and Coulomb term as the perturbation*

$$br + c > -\frac{4\alpha_{\text{s}}}{3r}$$

$$\rightarrow (br+c)^2 > \left(-\frac{4\alpha_{\text{s}}}{3r}\right)^2.$$

As seen earlier, from the above inequality we can get four perturbative cut-offs for the linear parent:

$$r_5^{\text{long}} = \frac{-c+\sqrt{c^2+(16\alpha_{\text{s}}/3)b}}{2b}, \tag{26}$$

$$r_6^{\text{long}} = \frac{-c-\sqrt{c^2+(16\alpha_{\text{s}}/3)b}}{2b}, \tag{27}$$

$$r_7^{\text{long}} = \frac{-c+\sqrt{c^2-(16\alpha_{\text{s}}/3)b}}{2b} \tag{28}$$

and

$$r_8^{\text{long}} = \frac{-c-\sqrt{c^2-(16\alpha_{\text{s}}/3)b}}{2b}. \tag{29}$$

Table 1. Possible r-values.

c (GeV)	Possible r^{short}'s	Possible r^{long}'s
1	$\frac{1+\sqrt{1-(16\alpha_s b/3)}}{2b}$, $\frac{1-\sqrt{1-(16\alpha_s b/3)}}{2b}$, $\frac{-1+\sqrt{1+(16\alpha_s b/3)}}{2b}$	$\frac{-1+\sqrt{1+(16\alpha_s b/3)}}{2b}$
$4\sqrt{\alpha_s b/3}$	$\sqrt{4\alpha_s/3b}$, $(\sqrt{2}-1)\sqrt{4\alpha_s/3b}$	$(\sqrt{2}-1)\sqrt{4\alpha_s/3b}$
0	$\sqrt{4\alpha_s/3b}$	$\sqrt{4\alpha_s/3b}$
$-4\sqrt{\alpha_s b/3}$	$(\sqrt{2}+1)\sqrt{4\alpha_s/3b}$	$(\sqrt{2}+1)\sqrt{4\alpha_s/3b}$, $\sqrt{4\alpha_s/3b}$
-1	$\frac{1+\sqrt{1+(16\alpha_s b/3)}}{2b}$	$\frac{1+\sqrt{1+(16\alpha_s b/3)}}{2b}$, $\frac{1+\sqrt{1-(16\alpha_s b/3)}}{2b}$, $\frac{1-\sqrt{1-(16\alpha_s b/3)}}{2b}$

Thus, we get four possible values of r^{short}'s (r_1^{short}, r_2^{short}, r_3^{short}, r_4^{short}) from Coulomb term as the parent and linear term as the perturbation and another four values of r^{long}'s (r_5^{long}, r_6^{long}, r_7^{long}, r_8^{long}) from linear term as the parent and Coulomb term as the perturbation.

From the positivity condition on r-values we can exclude r_4^{long} and r_6^{long}, leading to six possibilities: three each for r^{short} and r^{long} as given in table 1. Reality condition on these cut-offs ($c^2 \geq (16\alpha_s/3)b$) leads to two possibilities, $c > 4\sqrt{(\alpha_s/3)b}$ and $c > -4\sqrt{(\alpha_s/3)b}$, implying that c can be both +ve and −ve within this limit. In ref. [6] it is argued that $|c|$ should be less than 1 GeV. As we are dealing with mesons having reduced masses of only about 1 GeV or less, a value of c far exceeding this scale would presumably be unnatural.

Indeed there are specific phenomenological models [22,23], with $c = -0.5575, -0.6664, -0.82$, all less than 1 GeV.

From table 1, one can obtain specific values of r^{short} and r^{long} for representative values of $c = 1, 4\sqrt{\alpha_s b/3}, 0, -4\sqrt{\alpha_s b/3}, -1$ which are within the range $c \leq 1$ GeV.

Table 2. Possible r-values in Fermi.

Scale	c (GeV)	Possible r^{short}'s (Fermi)	Possible r^{long}'s (Fermi)
	1	0.9618, 0.1146, 0.0941	0.0941
	$4\sqrt{\alpha_s b/3}$	0.3320, 0.1375	0.1375
c-scale	0	0.3320	0.3320
	$-4\sqrt{\alpha_s b/3}$	0.8016	0.8016, 0.3320
	-1	1.1707	1.1707, 0.9618, 0.1146
	1	1.0152, 0.0612, 0.0549	0.0549
	$4\sqrt{\alpha_s b/3}$	0.2494, 0.1033	0.1033
b-scale	0	0.2494	0.2494
	$-4\sqrt{\alpha_s b/3}$	0.6021	0.6021, 0.2494
	-1	1.1314	1.1314, 1.0152, 0.0612

3. Results

In table 2, we record the numerical values of r^{short} and r^{long} at c-scale ($\alpha_s = 0.39$) and b-scale ($\alpha_s = 0.22$) with $b = 0.183$ GeV^2.

The above analysis shows that there are possibilities of overlapping or gap regions for Coulomb parent/perturbation and with the linear parent/perturbation.

Table 3. r^{short} and r^{long} in Fermi.

Scale	c (GeV)	$r^{\text{short}} = r^{\text{long}}$ (Fermi)
	1	0.0941
c-scale	$4\sqrt{\alpha_s b/3}$	0.1375
($\alpha_s = 0.39$)	0	0.3320
	$-4\sqrt{\alpha_s b/3}$	0.8016
	-1	1.1707
	1	0.0549
b-scale	$4\sqrt{\alpha_s b/3}$	0.1033
($\alpha_s = 0.22$)	0	0.2494
	$-4\sqrt{\alpha_s b/3}$	0.6021
	-1	1.1314

Table 4. r.m.s. radii with Coulomb term as the parent and linear term as the perturbation with different *c*-values.

c	α_s	Meson	r^{short} (Fermi)	$r_{\text{r.m.s.}}$ (Fermi)
$4\sqrt{\alpha_s b/3}$	0.39	$D(c\bar{u}/c\bar{d})$	0.1375	0.5691
		$D_s(c\bar{s})$		0.5751
		$J/\psi(c\bar{c})$		0.6033
	0.22	$B(\bar{b}c)$	0.1033	0.7045
		$\tau(b\bar{b})$		0.7453
0	0.39	$D(c\bar{u}/c\bar{d})$	0.3320	0.2428
		$D_s(c\bar{s})$		0.2481
		$J/\psi(c\bar{c})$		0.2748
	0.22	$B(\bar{b}c)$	0.2494	0.3066
		$\tau(b\bar{b})$		0.3479
$-4\sqrt{\alpha_s b/3}$	0.39	$D(c\bar{u}/c\bar{d})$	0.8016	0.0857
		$D_s(c\bar{s})$		0.0857
		$J/\psi(c\bar{c})$		0.0943
	0.22	$B(\bar{b}c)$	0.6021	0.0698
		$\tau(b\bar{b})$		0.0868

Table 6. Total r.m.s. radii for different *c* values.

			$r^{\text{total}}_{\text{r.m.s}}$ (Fermi)	
c	α_s	Meson	$r_0 = 7$ GeV^{-1}	$r_0 = 9$ GeV^{-1}
$4\sqrt{\alpha_s b/3}$	0.39	$D(c\bar{u}/c\bar{d})$	0.6910	0.6604
		$D_s(c\bar{s})$	0.6920	0.6627
		$J/\psi(c\bar{c})$	0.7080	0.6818
	0.22	$B(\bar{b}c)$	0.8191	0.7927
		$\tau(b\bar{b})$	0.8561	0.8306
0	0.39	$D(c\bar{u}/c\bar{d})$	0.3646	0.3341
		$D_s(c\bar{s})$	0.3760	0.3356
		$J/\psi(c\bar{c})$	0.3793	0.3532
	0.22	$B(\bar{b}c)$	0.4212	0.4002
		$\tau(b\bar{b})$	0.4585	0.4331
$-4\sqrt{\alpha_s b/3}$	0.39	$D(c\bar{u}/c\bar{d})$	0.2039	0.1760
		$D_s(c\bar{s})$	0.1990	0.1722
		$J/\psi(c\bar{c})$	0.1955	0.1716
	0.22	$B(\bar{b}c)$	0.1832	0.1575
		$\tau(b\bar{b})$	0.1953	0.1710

Therefore, unless they are identical (as in $c = 0$), the addition of two counterparts (linear part and Coulomb part), either overestimates or underestimates the calculated values of quantities which involves the integration over 0 to ∞. In such a case, the results should at best be interpreted as the upper bounds. On the other hand, if $r^{\text{short}} < r^{\text{long}}$ and there is a gap, then it will be interpreted as lower bound.

To obtain the most restrictive bounds, we have to choose the pairs of r^{short} and r^{long} when $|r^{\text{short}} - r^{\text{long}}|$ is minimum. This condition leads to the unique choice of pairs of r^{short} and r^{long}.

In table 3, we show the bounds on r^{short} and r^{long} in Fermi which yields exact/most restrictive upper bounds on the quantities to be calculated.

The application of Airy function as meson wave function needs suitable cut-off to make the analysis normalizable and convergent. We therefore set the cut-off in the range 7–9 GeV^{-1} (equivalently 1.379–1.773 Fermi) for our calculations [10].

Table 5. r.m.s. radii with linear term as the parent and Coulomb term as the perturbation with cut-off 7 and 9 GeV^{-1} for different *c* values.

				$r_{\text{r.m.s.}}$ (Fermi)	
c	α_s	Meson	r^{long} (Fermi)	$r_0 = 7$ GeV^{-1}	$r_0 = 9$ GeV^{-1}
$4\sqrt{\alpha_s b/3}$	0.39	$D(c\bar{u}/c\bar{d})$		0.1219	0.0913
		$D_s(c\bar{s})$	0.1375	0.1168	0.0875
		$J/\psi(c\bar{c})$		0.1046	0.0784
	0.22	$B(\bar{b}c)$	0.1033	0.1146	0.0881
		$\tau(b\bar{b})$		0.1107	0.0852
0	0.39	$D(c\bar{u}/c\bar{d})$		0.1218	0.0912
		$D_s(c\bar{s})$	0.3320	0.1278	0.0875
		$J/\psi(c\bar{c})$		0.1046	0.0784
	0.22	$B(\bar{b}c)$	0.2494	0.1146	0.0935
		$\tau(b\bar{b})$		0.1106	0.0851
$-4\sqrt{\alpha_s b/3}$	0.39	$D(c\bar{u}/c\bar{d})$		0.1181	0.0903
		$D_s(c\bar{s})$	0.8017	0.1132	0.0865
		$J/\psi(c\bar{c})$		0.1011	0.0772
	0.22	$B(\bar{b}c)$	0.6021	0.1133	0.0876
		$\tau(b\bar{b})$		0.1084	0.0841

Table 7. $r_{\text{r.m.s.}}$ values in Fermi.

	$r_{\text{r.m.s.}}$ (Fermi)		
Meson	Ref. [24]	Ref. [25]	Ref. [26]
$c\bar{c}$	0.4490	0.4453	0.4839
$b\bar{b}$	0.2249	0.2211	0.2671

In tables 4, 5 and 6, we summarize the results for r.m.s. radius with $c = 4\sqrt{\alpha_s b/3}$, 0, $-4\sqrt{\alpha_s b/3}$ for Coulomb term as the parent and linear term as the perturbation and linear term as the parent and Coulomb term as the perturbation and the total r.m.s. radii respectively.

The input parameters in the numerical calculations used are $m_u = 0.336$ GeV, $m_s = 0.483$ GeV, $m_c = 1.55$ GeV and $m_b = 4.95$ GeV, $b = 0.183$ GeV^2.

Now from tables 4 and 5, we can find the total r.m.s. radii as shown in table 5.

Now for comparison, in table 7, we give the model predictions of r.m.s. radii for heavy flavoured mesons available in literature.

Comparing table 6 with available model predictions (table 7) for $c\bar{c}$ and $b\bar{b}$ mesons, we can conclude that r.m.s. radii with $c = 4\sqrt{\alpha_s b/3}$ (table 5) are higher than the predicted results [24–26], while with $c = -4\sqrt{\alpha_s b/3}$ the radii are too low for $c\bar{c}$ but close for $b\bar{b}$ meson. With $c = 0$, the results are close to table 6 for $c\bar{c}$ meson but doubles for $b\bar{b}$ meson.

It is to be noted that r^{short} and r^{long} are the perturbative saturation lengths for the Coulomb parent and the linear parent. The proper perturbative range should be far less than r^{short} and r^{long}. As a consequence, our results should, at best, be considered as the lower bounds as the estimated values should exceed the real ones. The prediction with $c = 4\sqrt{\alpha_s b/3}$ conforms to this expectation. Our analysis thus indicates $c = 4\sqrt{\alpha_s b/3}$ to be the phenomenologically preferred value as far as the present version of the model is concerned.

4. Conclusions

In this paper, we have reported a reformulation of the QCD potential model [6–11] where quantum mechanical perturbation methods [13,14] are used. The present version takes care of the following two aspects of quantum mechanics:

(a) The scale factor c in the potential should not affect the wave function of the system even while applying the perturbation theory.

(b) Choice of perturbative piece of the Hamiltonian (confinement or linear) should determine the effective radial separation between the quarks and the antiquarks.

It has been shown that in spite of the above two quantum mechanical constraints, the scale factor c of the potential appears in defining the finite perturbative compatible ranges r^{short} and r^{long} of interquark separation. In the earlier version of the model, both r^{short} and r^{long} were assumed to be identical and of infinite length. We have then used it to calculate the root mean square radii of various heavy flavoured mesons and compared these with predictions of available theoretical models. The value $c = 4\sqrt{\alpha_s b/3}$ seems to be the optimum choice.

Acknowledgements

DKC thanks Prof. N D Hari Dass of Chennai Mathematical Institute (CMI) for bringing his attention to the basic limitation of the earlier version of the model, while TD acknowledges the support of University Grants Commission in terms of fellowship under BSR scheme to pursue research at Department of Physics, Gauhati University, India.

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